PROBLEMS AND SOLUTIONS

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Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal, nor posted to the internet before the due date for solutions. Submitted solutions should arrive before August 31, 2015. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11831. *Proposed by Raitis Ozols, University of Latvia, Riga, Latvia.* Prove that for $\varepsilon > 0$ there exists an integer n such that the greatest prime divisor of n^2+1 is less than εn.

11832. *Proposed by Donald Knuth, Stanford University, Stanford, CA.* Let $C(z) = \sum_{n=0}^{\infty} \binom{2n}{n} \frac{z^n}{n+1}$ (thus $C(z)$ is the generating function of the Catalan numbers). Prove that

$$\log(C(z))^2 = \sum_{n=1}^{\infty} \binom{2n}{n} (H_{2n-1} - H_n) \frac{z^n}{n}.$$

Here, $H_k = \sum_{j=1}^{k} 1/j$; that is, H_k is the kth *harmonic number*.

11833. *Proposed by Mher Safaryan, Yerevan State University, Yerevan, Armenia, and Vahagn Aslanyan, University of Oxford, Oxford, U. K.* Let f be a real-valued function on an open interval (a, b) such that the one-sided limits $\lim_{t\to x^-} f(t)$ and $\lim_{t\to x^+} f(t)$ exist and are finite for all x in (a, b). Can the set of discontinuities of f be uncountable?

11834. *Proposed by Arkady Alt, San Jose, CA.* For nonnegative real numbers u, v, w, let $\Delta(u, v, w) = 2(uv + vw + wu) - (u^2 + v^2 + w^2)$. Say that two lists (a, b, c) and (x, y, z) *agree in order* if $(a-b)(x-y) \ge 0$, $(b-c)(y-z) \ge 0$, and $(c-a)(z-x) \ge 0$. Prove that if (x, y, z) and (a, b, c) agree in order, then $\Delta(a, b, c)\Delta(x, y, z) \ge 3\Delta(ax, by, cz)$.

11835. *Proposed by George Stoica, University of New Brunswick, St John, Canada.* Find all functions f from $[0, \infty)$ to $[0, \infty)$ such that whenever $x, y \ge 0$,

$$\sqrt{3} f(2x) + 5 f(2y) \le 2 f(\sqrt{3}x + 5y).$$

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